MATHEMATICS TEACHING

THE BULLETIN OF THE

ASSOCIATION for TEACHING AIDS in MATHEMATICS

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THE BULLETIN OF THE
ASSOCIATION FOR TEACHING AIDS IN MATHEMATICS

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EDITORIAL

The following pages show a rapidly increasing membership and a variety of activities by the Association in many parts of the country. A most encouraging result of the recent meeting at Blackpool was the decision to form a local group of members in the Lancashire area. Mr. Birtwistle, of 1 Meredith Street, Nelson, Lancs., will be the convenor of this group and he will be pleased to give further details to anyone interested in joining in its work.

Among the future meetings which the Association is arranging (or helping to arrange) are meetings in London and Cambridge devoted to Modern and Primary Schools respectively. We have been dissatisfied with the comparatively small amount of work which we have done in these fields hitherto and it is hoped that these conferences will enable us to expand it. The pages of this journal are always open to writers on Modern and Primary work, but those best qualified to write on it—that is those actually engaged on the task—are excessively modest and reluctant to share their best ideas. This issue contains some material which we hope will interest Modern School teachers. Can we have more, and can we have some articles on teaching the beginnings of the subject in Primary School? If you have views on the subject but do not feel that they will make an article why not write a letter? We have not so far run a correspondence column; who will begin it?

WANTED: MORE MATHEMATICIANS

During the last few months a great deal has been written, and much more has been said, about the shortage of mathematicians and, consequently about the teaching of mathematics. In general, the problem is presented in such a way as to consider the mathematician as a fundamental necessity: this is a scientific age and therefore we need more scientists, whose training must necessarily have a mathematical foundation. It seems a pity that a mathematician should be regarded purely as a means to an end; namely, more scientists. The fact remains, however, that mathematicians are becoming more unobtainable and so the teaching of mathematics has been subjected to strong criticism. Is this criticism fully justified? In some cases the answer is undoubtedly "Yes", but very few of the reports mention the improvements in the teaching of mathematics.

In recent years the subject has become much more "alive" in many of our schools by the use of film-strips and models, and a deeper insight is given into its practical applications.

One of the largest stumbling-blocks is, perhaps, the examination system. The teacher works with classes which are often too large and with the knowledge that a syllabus must be completed. With large numbers and limited time it is often difficult to present mathematics as a growing subject: one needs time to show how our present knowledge has developed and in what ways it is still growing. Without such knowledge, no child can be expected to grow into a true mathematician. At least one report has said that girls are frightened of the subject: they would have no reason for this fear if they understood the kind of thought and logic which has made mathematics the most perfect of the sciences.

In a recent report instituted by the Carnegie Corporation in New York, it is suggested that children are afraid of mathematics because of "nagging parents or the snowballing discouragement that comes of always getting wrong answers." The first of these causes can hardly be remedied by the teacher who, all too often, hears the same parental comment: "maths is so important." The teacher can, however, do something about the second cause. No conscientious teacher will ignore the child who persistently gets the wrong answer; and a little extra attention (assuming that the class is not an unreasonably large one) will lead to the root of the trouble. Probably, as in too many cases, the child cannot multiply correctly. Learning multiplication tables seems to be an "old-fashioned" idea; but is it so unwise? Obviously such tables should not be presented only at their face value, but once a child has understood why $6 \times 7 = 42$ there can be no harm in him learning it as a fact to be used automatically for the rest of his mathematical career.

Another American criticism which has its parallel in our own schools is that junior mathematics, and particularly elementary arithmetic, is frequently taught by teachers whose own mathematical grounding is strictly limited. In the recent survey of mathematics in training colleges, it was pointed out that only a small percentage of our training colleges have trained mathematicians on the staff. The inevitable result is that elementary arithmetic and some junior mathematics is being taught by people whose basic feeling is a horror of mathematics. Without previous contact with a trained mathematician, the prospective teacher finds he is fighting against communicating his fear of the subject to his pupils.

It is in the junior schools that the foundations of mathematics are laid; but not all primary school teachers have sufficient mathematical learning to be able to teach the conception of number. This does not mean that the child entering a secondary school is unable to cope with elementary arithmetic. The intelligent ones have learned how to get the right answer, but have they learned how to think mathematically?

It may be that we expect too much of even the most intelligent children. In one of the many reports it has been pointed out that the apparently elementary ideas of number came late to civilisation. The decimal point is less than 100 years old, but an 11-year-old child is expected to have a complete understanding of the decimal system. Perhaps if a longer period of learning were devoted to the basic conceptions of number we would eventually see a generation of aspiring mathematicians.

Whichever stage of education one considers, the plea is always the same: "We must have more teachers who are mathematicians."

Could it be that this scientific age attaches such vital importance to the scientist, that the majority of mathematical brains are steered into channels which produce scientists, and not mathematicians? If this is so then the scientific age will defeat its own ends.

I. L. CAMPBELL.

When you mention statistics these days, people always think you mean a set of three figures to describe one figure!

THE DUKE OF EDINBURGH.

THE INTRODUCTION, USE AND FUTURE OF NUMERICAL AND PRACTICAL METHODS OF MATHEMATICS

A. J. WALKER

If we consider either the historical introduction of mathematics or the mathematics syllabus as presented in a school, we notice that the preliminary ideas are centred about the manipulation of numbers and we usually give the name of arithmetic to this particular branch of study. After a time we find that a number of calculations can be generalised by introducing letters or symbols to represent any number, operating on these symbols with certain laws and then by inserting numbers for the symbols we can transform our calculation into an enumerable set of particular results. As a student of mathematics progresses he gains great satisfaction from generalisations of this nature and indeed probably feels that he is only being truly mathematical when dealing with these generalised forms. As one progresses further, however, one finds that theoretical solutions become more and more difficult until there comes a time when no known method is applicable for obtaining a solution. For example, one can solve equations of the form $f(x, a)=0$; $f(x^2, x, a)=0$; $f(x^3, x^2, x, a)=0$; $f(x^4, x^3, x^2, x, a)=0$; but when it comes to equations of the form $f(x^5, x^4, x^3, x^2, x, a)$ and linear functions of higher powers of x we find that in general there are no theoretical methods of solution.

If all mathematics was "Pure Mathematics," then perhaps we could continue with our research and analysis without any specific solutions at all, but the fact remains that the usefulness of mathematics depends upon its ability to answer problems in applied mathematics and this we cannot do unless we can obtain solutions to our theoretical mathematics. We come to a stage, therefore, when although we can advance in theory these advances are of little use to us if we cannot obtain any solutions to our theoretical calculations in practice. To overcome this stumbling block in the development of mathematics we retrace our footsteps so as to speak and direct our attention to a numerical method giving a particular solution rather than a general one.

Our first contact with a numerical method, apart from the obvious numerical calculations in arithmetic, appears when we consider graphical methods of solution of equations. We take, for example, an equation—say a quadratic function equated to zero—and by inserting particular values for the variable we can obtain a graph of the function which will show us the general picture of the function together with an approximation to the solution of the equation. Similarly for the solution of higher order algebraic equations and differential equations our preliminary study is a graphical one.

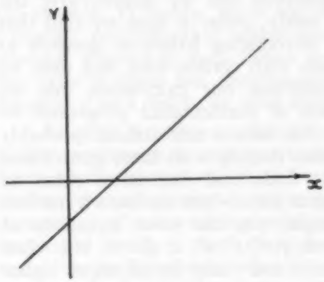
Numerical methods can, broadly speaking, be divided into three sections, namely: —

(i) Graphical methods, *i.e.* methods which give an overall picture of the function with perhaps (though not in general) an approximate solution.

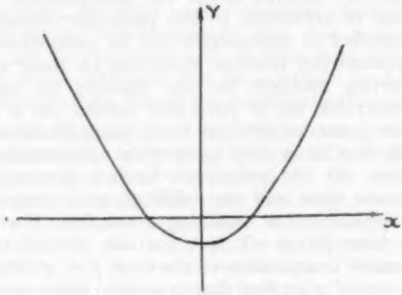
(ii) Iterative procedures or methods of successive approximation.

(iii) Interpolative procedures, *i.e.* methods which build up a solution over a whole range from a few preliminary values.

The graphical methods at our disposal vary very much according to the problem under examination. In the simplest case it may be possible to plot just a few values of a function and join these up with a smooth curve (*i.e.* in the case of simple "well-behaved", continuous functions). For example $y=x-1$ or $y=x^2-1$

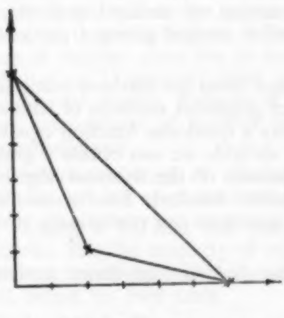


$y=x-1$

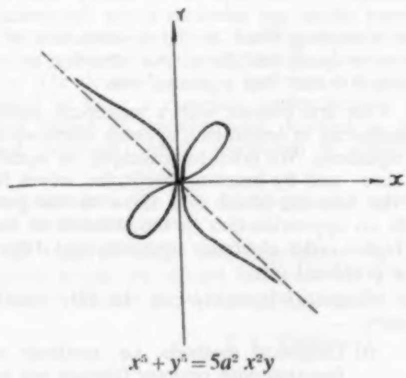


$y=x^2-1$

On the other hand, it may be necessary to use such devices as Newton's Diagram. For example:— $x^5+y^5=5a^2x^2y$.



NEWTON'S DIAGRAM

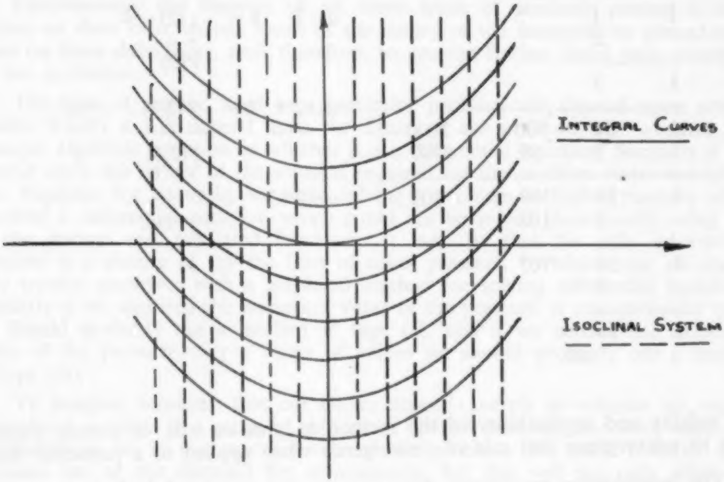


$x^5+y^5=5a^2\,x^2y$

In the case of differential equations we require an elaborate graph containing such features as isoclinal systems, integral curves, discriminant loci, tac loci, cusp loci and envelopes. For example $\frac{dy}{dx}=x$.

Isoclinal system given by:

$x=$constant.



With iterative procedures the general principle underlying all the different methods is the successive way in which the method, when applied to a particular problem, approaches more and more closely to the true solution starting from an approximated value. For example solving $x^3-3x^2+5x-8=0$ by Horner's or Newton's method. One root lies between 2 and 3 since $f(2)=-2$ and $f(3)=7$.

Where $f(x)=x^3-3x^2+5x-8$.

Horner's Method

2·	1	−3	5	−8
		2	−2	6
	1	−1	3	−2
		2	2	
	1	1	5	
		2		
	1	3		
2·3	1	30	500	−2000
		3	99	1797
	1	33	599	−203
		3	108	
	1	36	707	
		3		
	1	39		

etc.

Newton's Method

If a represents approximation then error $h=\dfrac{-f(a)}{f'(a)}$

$a=2$, $h=\dfrac{-f(2)}{f'(2)}=\cdot 4$

$a=2\cdot 4$, $h=\dfrac{-f(2\cdot 4)}{f'(2\cdot 4)}=-\cdot 07$

$a=2\cdot 33$

etc.

The validity and applicability of the method in practice will, of course, depend upon its convergence and rate of convergence when applied to a particular case.

The interpolative procedures, however, are the most useful of the numerical methods in practice, since the majority of the work using such methods of solution requires a whole range of tabulation. It is methods of this type which are used almost exclusively for the numerical solution of differential equations. The method depends upon the continuous prediction of the next value from say four previous ones as we progress across the range of solution. For example in the solution of $\dfrac{dy}{dx}=y+x^2$ such that $x=0$ when $y=1$, we first calculate the value of y for say $x=0\cdot 1$, $0\cdot 2$, $0\cdot 3$ by using the Taylor expansion, and form a "Difference" table.

x	y	$q=\dfrac{1}{n}\dfrac{dy}{dx}=(y+x^2)/10$	Δq	$\Delta^2 q$ etc.
0·0	1·00000	0·100000	·011551	
·1	1·10551	0·111551	·014870	(For error
·2	1·22421	0·126421	·018537	estimation).
·3	1·35958	0·144958	·022590	

We then predict $y_{\cdot 4}$ by using Milne's formula: $y_1=y_{-3}+\cdot 4(2q_{-2}-q_{-1}+2q_0)/3$

$$y_{\cdot 4} = 1{\cdot}00000 + 4({\cdot}223102 - {\cdot}126421 + {\cdot}289916)/3$$
$$\therefore \quad y_{\cdot 4} = 1{\cdot}51546 \quad \text{hence } q_{\cdot 4} = {\cdot}167546.$$

We check this with Simpson's formula: $y_1 = y_{-1} + \frac{1}{3}(q_{-1} + 4q_0 + q_1)$

$$y_{\cdot 4} = 1{\cdot}22421 + \tfrac{1}{3}({\cdot}126421 + {\cdot}579832 + {\cdot}167546).$$
$$\therefore \quad y_{\cdot 4} = 1{\cdot}51548 \quad \text{and } q_{\cdot 4} = {\cdot}167548.$$

We fit this last value into the difference table, complete the differences, and then repeat for $y_{\cdot 5}$ etc.

Unfortunately the theories of all three types of methods present a large subject on their own, indeed much of the theory of the interpolative procedure is based on finite differences, and, therefore, to attempt further detail here would be far too ambitious.

The type of method used in a particular problem will depend upon several factors. Firstly it will depend upon the nature of the problem, *e.g.* whether it is a simple algebraic equation or whether it is a differential equation. Secondly it will depend upon the nature of the solution required for the problem under consideration. Suppose, for example, we were solving a problem in hydrodynamics which involved a differential equation which could not be solved theoretically using any of the known and tabulated functions. It may be that the only information required is a picture of say the lines of equal pressure. In which case we should only trouble ourselves with a graphical method for solving differential equations. Similarly if we required the numerical value of the pressure at *one* particular point we should probably use a method of type (ii), and if we needed the numerical value of the pressure over a range of values we should probably use a method of type (iii).

To imagine, however, that our choice depends merely on whether we require a graphical, a single or a range of solution is wishful thinking. The more important considerations lie in the practical side of the problem. We must consider what facilities are at our disposal for computation, for this will not only affect our accuracy but will decide whether or not a method is practicable. Let us assume that our computational facilities amount to a pen, paper, a human brain and the use of sufficiently accurate mathematical tables. At first sight it would appear that under these conditions the theoretically shortest numerical method would be the one to use, but we must be careful to consider the nature of the calculations involved. In theory for example the cubing of a number is no more nor less complicated than the squaring, but practically speaking the extra work involved may be very great. We must also bear in mind that the more complicated arithmetical concepts used, the greater is the need for imposing checks.

The question of checking is very important indeed when we are considering practical mathematics. In algebra we do not usually bother about checking until we have obtained our solution, and then if on inserting our solution back in the original equation we do not obtain an identity, we merely check through our previous work, correcting where necessary. How different is the situation when we are computing numerical methods. Firstly, as in solving differential equations, we cannot, generally speaking, check our solution by obtaining an identity in the original equation since the work involved is often as great as that required for

obtaining the solution, and may lead to further error. Secondly, due to the amount of arithmetic which may be involved we may spend hours of wasted time on useless calculation if an error were made right at the start of the calculation. We attempt, therefore, to use a method of checking by which we can ascertain the correctness, or otherwise, of our work continuously throughout the whole calculation. There are many ways in which this can be done, but perhaps the most common is known as cross-checking whereby calculations are made in two different ways concurrently and the two answers compared: *e.g.* If we have to multiply a column of figures $\mathbf{x}$ by the same multiplier M we say

$$\Sigma M\mathbf{x}=M\Sigma\mathbf{x}$$

In this way we make sure that only a small portion of the calculation has to be repeated.

So far then our decision as to which numerical procedure to use will depend upon: —

(a) Nature of problem (i) Qualitative
(ii) Quantitative

(b) Analysis of arithmetical procedure involved by the different methods.

(c) Analysis of the suitability for providing checks.

The method which conforms most effectively with these requirements is usually chosen.

Using computational facilities as afforded above, however, we find that apart from solutions of simple algebraic equations of a special type, the numerical methods become not only tedious and cumbersome but in some cases quite impracticable. The reason for this lies in the fact that either the rate of convergence to the solution is very slow or the interval at which we have to work is extremely small. Fortunately, man is not dependent upon purely manual labour even when it comes to calculation, for he has at his disposal many types of calculating machines.

These machines are divided into three main groups: —

(a) Desk machines
(b) Accounting machines
(c) Installations.

We are not here concerned with accounting machines since they are designed for a particular form of calculation as suggested by their name. The other two groups can be further sub-divided as follows: —

Desk		*Installations*		
Hand	*Electric*	*Punched Cards*	*Analogue machines*	*Automatic computers*
			Differential analysers	

The operations performed by these machines are nothing more nor less than those basic operations of arithmetic, namely, addition, subtraction, multiplication and division; indeed, as far as desk machines are concerned, they are only a means by which we can perform the individual sums more easily and quickly, *i.e.* instead of doing a multiplication sum on paper we do it on the machine. The

difference in operation of hand and electrical machines is nothing fundamental but just a difference in speed of operation, the electrical machine being the faster. An experienced operator of these desk machines will, of course, employ short cuts in the calculation by various tricks of the trade, but we must remember that the more involved or complicated an individual step may be the greater is the likelihood of an error being made, and in consequence this may mean an increase in the amount of checking necessary. Thus by employing a desk machine we are only increasing our rate of calculation and we present our numerical method in exactly the same form as for the wholly manual form of calculation. In practice the individual calculations are set out systematically by a mathematician and the actual computation is carried out by a skilled machine operator, who merely adds, subtracts, multiplies or divides the numbers as directed, tabulating the answers. For example, suppose we want to find the mean and variance of a set of statistical data.

Observation x_i	14	24	34	44	54	64	74	84	94	104	114
Frequency f_i	1	2	7	20	24	31	38	24	21	7	3

In order to find the mean and variance we must use the following formulæ:—

$$\text{Mean } \bar{x}=\frac{1}{n}\Sigma f_i x_i \qquad \text{where } n=\Sigma f_i$$

$$\text{Variance } s^2=\frac{1}{n}\Sigma x_i^2 - (\bar{x})^2$$

The mathematician splits these formulæ into steps for the computer.

(i) Σf_i —record.

(ii) $\Sigma f_i x_i$ —leave on machine.

(iii) Divide $\Sigma f_i x_i$ by Σf_i—record $(\bar{x})$

(iv) Calculate column of x_i^2—record.

(v) $\Sigma f_i x_i^2$—leave on machine.

(vi) Divide $\Sigma f_i x_i^2$ by Σf_i —record $(\frac{1}{n}\Sigma x_i^2)$

(vii) Calculate $(\bar{x})^2$.—record.

(viii) Subtract $(x)^2$ from $\frac{1}{n}\Sigma x_i^2$ —record

The "punched card" installation is generally used in conjunction with the other installations and not as a calculator on its own, and I shall, therefore, not discuss its particular contribution.

The Analogue machine, as the name implies, uses physical quantities such as length or electric charge and solves an analogous physical problem to the one under examination. The solutions obtained are generally of a graphical nature, and a machine of this type is used extensively as a differential analyser, *i.e.* used for producing graphical representations of differential equations. Since the operation and indeed the working of an analogue machine is so different from the other types, I do not propose to discuss it further.

The automatic computing machine is again divisible into two main classes: those working on electric relays, and those working on electronic valves. In general

principle there is nothing to choose between them (as in the case of the hand and electric desk machines), but the rate of operation of the electronic machine is several times greater than that of the relay machine, and in turn the relay machine is hundreds of times faster than the desk machines.

Mathematically speaking, the only difference between these types of computers and the desk models is that they use the binary scale as opposed to the decimal system in the more common machines. The reason for this is that the valves or relays either possess an impulse or they do not, *i.e.* two alternatives and not ten as required by the decimal system. This difference presents no practical difficulty since we only need an additional unit connected to the machine to convert one system into the other and vice-versa.

As in the case of the desk machines I shall not go into the mechanics of these machines but limit myself to a very brief discussion on the use of the machine from the mathematician's point of view. The first requirement, as we have seen before, is to investigate all the aspects of the particular calculation we are about to perform, and because of the more complex problems involved when making use of these machines, we consider: —

(i) Mathematical analysis.
(ii) Arithmetical procedure.
(iii) Error analysis.
(iv) Practical computing procedure.
(v) Checking procedure.
(vi) Layout.
(vii) Checking final work.

Our investigation is no longer so much concerned with length of methods nor the total time taken by the individual calculations since the time taken by the machine is so small. What, then, are the principal considerations in the above list? The answer to this question lies in the direction of the machine, *i.e.* how we instruct the machine to perform its various tasks. Contrary to certain schools of public opinion, an automatic computer—sometimes called an electronic brain—has no power of thought of its own. All it can do is to carry out numerous instructions (all of which have been reduced to basic operations of arithmetic by a mathematician) very quickly indeed or else to select, by a fantastic arrangement of "trial and error" selections, with checking, the correct answer from one of its stores. In other words, the whole operation is one of directed, systematic repetition of certain fundamental processes.

The instructions are fed into the machine by cards with holes punched in them, which tell the machine by means of a coded system to add say, the number from store 3 to the number in store 107, to square the sum and to put the answer in store 118. The stores of these machines consist of delay lines together with various sorts of physical and magnetic means of storage, where numbers or instructions can be filed away, so to speak, until required by the machine. We must feed into the machine the numbers necessary for our calculation and the instructions for both direct calculation and checking procedure. The last of these must be of such a nature as to give a continuous check throughout the whole calculation. We see, therefore, that to ascertain the merits or otherwise of numerical

methods when used in conjunction with automatic computers is a very complex problem indeed, and as with some of the previous topics a complete discussion is not possible here.

Even for the simplest calculations the work needed for compiling the operation technique for the machine is very complicated. The importance of the work, however, is that once we have made up our system of operation we can perform the calculation for an infinity of related problems. We find, too, that by taking different combinations of different instructions we can build up other processes for solving equations in other fields with very little additional work. Another important point to consider is that when using iterative procedures we are constantly performing the same operation again and again. Hence by making the machine repeat its instructions on a new number (a simple instruction in practice) we can solve even the most slowly converging problem with no further trouble than for a quickly converging one.

The limit to what these machines can do, assuming that an arithmetical procedure exists, is dependent only upon their size and indeed on the ability of the mathematician to use their resources to the full. It is also this limit of size which determines the complexity of the instructions required—the bigger the machine the simpler the instructions.

With the ever-increasing use and production of these machines, together with the increasing size, we see that a whole new branch of mathematics is within our reach. This mathematics is not new in the sense that it brings in new fundamental ideas or indeed new discoveries of a revolutionary nature, but it does mean that mathematics can have a far greater field of application. Hitherto the field of mathematical physics which could be used for obtaining numerical results was narrowly restricted by the few functions which had been tabulated by pure mathematicians. Even some of the most common differential equations could only be solved in a few special cases, and even then boundary conditions had often to conform to a set pattern. With numerical methods combined with the appropriate facilities for their computation, however, this restriction is lifted.

In the above account I have tried to introduce the concept of numerical methods as necessitated by the need for solutions to problems, together with a discussion as to the practical computation of these methods. The studies in numerical methods are comparatively recent and there is every reason to suppose, with the ever increasing use of these methods as facilitated by the advent of the automatic computing machine, that a wider acceptance and use will be made of numerical analysis. I have omitted many aspects of the subject, such as the work on the analysis of errors (work which enables correct solutions to be obtained even though an error has been made during the calculation—based on the theory of finite differences) due to the necessary limits of available space, nevertheless there is still a great deal of research to be done in both numerical and practical mathematics. This is reflected by the fact that with few exceptions (I know of only one) this branch of mathematics is not generally taught in Universities, and in consequence very few students are prepared for research in this direction.

THE STUDENT'S PASSIVE ATTITUDE TOWARDS MATHEMATICS AND HIS ACTIVITIES

RUBEN SCHRAMM

At probably all levels, the non-professional study of mathematics is characterised by a marked passivity of the students. Without guidance, pupils will solve only such problems as can be fitted into some already familiar pattern. The formulation of problems in mathematical terms constitutes a major difficulty too. It is the object of this paper to characterise this attitude, trace its causes and point out ways to overcome it.

The solution of problems posed to students is either evaded altogether, or consists in unco-ordinated moves instead of a succession of steps directed by the pupil's innermost personality, as it should be. Instruction has improved during the last fifty years but not to the extent of converting the subject into a popular pastime. Its appeal is as yet insufficient to express itself in independent research, conducted just for the fun of it. Rather, it is accompanied by a certain uneasiness which is also to blame for the small amount of transfer of learning to new mathematical and extra-mathematical situations. Unpleasant subjects tend to be mentally isolated just to avoid their spreading. Therefore, a desirable goal would be to smooth mathematical activity and remove mental barriers. Utilitarian considerations demand this too, and in an age of constantly changing technical and scientific methods the relevant methematical methods undergo similar alterations. Under such circumstances, a study of any set of standard theories only, without a creative approach, is insufficient for the mastership of advanced techniques. Adaptability is the order of the day.

(a) *Characterisation of Mathematical Passivity.*

Passivity is the outward manifestation of a too strong reliance on authority which is exemplified either by individuals (teacher, parent, brother, friend) or by "infallible authorities" in the mathematical theories themselves, e.g., formulas and in more serious cases even by special categories of formulas only (usually acquired at an earlier stage (with which the pupil feels more secure than with others (the newer ones). This dependence results either in no solution at all (the worst case), in a very clumsy one (using only "archaic," i.e., secure methods—the best case), or in a confused attempt at a solution, based on fragments of relatively independent methods.

This characterisation is borne out by an analysis of students' mistakes.

(b) *Some Causes of Mathematical Passivity.*

The danger of falling into an attitude of dependency is inherent in every problematic mathematical situation because such a situation refers the pupil back to general theorems "governing" the case in point. Since the act of selection is usually not presented explicitly as such to the pupil, there ensues—instead of the theorems being properly arranged in his mind—a reverse process of "arrangement" of the mind through them, in their capacity as authorities. On the one hand, the pupil emerges with the impression that mathematics is identical with a constant substitution in general formulas, while on the other hand the attempt to follow that impression does not bring him any nearer to practical solutions. He suspects the existence

of something else necessary, and since the teacher (usually) solves problems correctly, this something appears as some witchcraft, deliberately withheld from him. Lengthy chains of argumentation at advanced levels of learning tend to confirm that impression still more by posing the question: "What holds together all the 'unrelated' steps and how can the teacher memorise them?" As a consequence, pupils cling even more desperately to formulas, using them indiscriminately and irrespective of relevance. (Time and again pupils overlook solutions obtainable by inspection, operating instead a ponderous apparatus of successive formulas, just because this can be done by a general memorised method, and does not entail direct contact with the mathematical reality in point.)

(c) *How can the pupil achieve an Active Attitude in Mathematical Work?*

Not all the remedies are directly available to the teacher. Two main points should serve as directives:

1. Those handicapping emotional influences which have created the authority complex have to be eliminated or, at least, weakened. The pupil has to be convinced constantly by every possible means that the teacher's success at achieving solutions is due exclusively to free association of relevant mathematical theorems which functions so freely just because of the satisfaction which he derives from it. Therefore, it can be achieved by the student too, at least the normal one, provided that the teacher can "infect" him with his own joy in mathematical work. This should not prove to be too difficult.

The pupil's conviction with regard to this character of mathematical work must be instinctive rather than conscious. It will definitely not be achieved by sporadic "lectures," only by constant explanation of the "key-ideas" governing proofs and the minimisation of the "teacher-mystery" as far as possible.

The external arrangements of formulas (in writing) must follow naturally from the inner architecture of ideas. Therefore they should not be rigid or another distracting factor will be introduced. Pupils should be encouraged to analyse problems on the merits of the case, just as in similar circumstances in life, without forcibly fitting them into some prefabricated pattern.

2. While during the actual work only free association can guarantee the discovery of ways of solution, the actual emergence of the right associations can be induced in advance and it is exactly this inducement which constitutes the act of instruction. In conducting this process, the "bridges of thought" most frequently employed in different kinds of mathematical work have to be consolidated so as to strike the eye immediately whenever their use could be of advantage. One of the primary criteria of syllabus organisation should thus be the difficulty of the general modes of thought involved in solving problems (degree of generality of abstractions used, difficulty of indirect reasonings, complexity of configurations to be combined, etc.). The character of the progress made in that direction should be expressly imparted to the pupil so as to bring the real difficulties into focus.

Besides becoming familiar with these general modes of thought the pupil has to learn and ultimately take for granted that when dealing with different mathematical theories he has to consider different "key-questions." Since in any theory the only particular means at his disposal are the theorems, he should become

accustomed to applying himself to that specific aspect of them which he needs while constructing proofs for posed problems. That means that theorems should systematically be grouped with regard to their results (this should be actually carried out). For example, all theorems in geometry having as their result the equality of two angles will constitute one of the groups. Therefore, when in some particular case the equality of two angles has to be proved, an inspection of the theorems constituting that group immediately induces typical questions referring either to the congruence of suitable triangles, the existence of isosceles triangles in the configuration or the equality of subtended arcs, etc., since only theorems of that group could furnish the desired result directly. (On the other hand, while performing similar problems by vectors other questions would have been typical. The pupil should therefore shift easily from groups of questions peculiar to one theory to those peculiar to another.)

While this method provides useful practice in the construction of means to given ends, the following complementary method stresses the process of derivation itself by avoiding any definite goal stated in advance. A certain collection of data, (e.g., some specific geometric configuration) is assigned to the student, but no definite proposition to be reached is stated. The pupil is told to draw all conclusions he observes and is capable of deriving until all possibilities seem exhausted. This method does not restrain his initiative. He has no reason to suppress results just because they might be no good for the proof of some teacher-imposed theorem.

Finally, a different attitude should be adopted towards the pupils' mistakes which should be regarded as products of his mental activity just as his correct statements. A thorough analysis of their nature can be a powerful means for the detection of handicapping tendencies. With a view towards the elimination of mistakes, such analysis should gradually be transferred to the student. He should recognise the non-accidental, deep-rooted character of his mistakes by noting repetitions. For example, some very prevalent mistakes merit the name of "analogy-mistakes" because they result from the application of a rule appropriate to a situation which bears only an outward resemblance to the case in point.* Pupils should understand that this error of judgment is due to a tendency of surrendering to authorities—the analogous formulas in the case discussed—before making sure of their actual relevance. Understanding their erroneous tendencies will help students in their elimination, but the teacher should discuss presented solutions in their entirety, so that the pupil recognises the good merits of his work too.

*** Because the nth derivative of $\sin x$ is $\sin(x + \frac{1}{2}n\pi)$ students may say that the nth derivative of $\sinh x$ is $\sinh(x + \frac{1}{2}n\pi)$.**

A DOMINO PROBLEM

The ordinary set of dominoes consists of 28 dominoes numbered from double-nought to double-six. Any child who has played with them will tell you that they can be arranged with adjacent halves matching to form a closed chain.

The problem is to prove that the same is true of any set of dominoes running from double-nought to double-n when n is even, but not when n is odd.

MATHEMATICS IN THE SECONDARY SCHOOL CURRICULUM

II—Mathematics is a Discipline of the Mind

R. H. Collins

In the first of these articles we considered the criteria which should be used to examine the aims for teaching mathematics (as distinct from arithmetic or number operation) in the Secondary School. It was stressed that if the subject was to continue to have such an important allocation of teaching time it must be shown that there are good reasons for teaching it to all children, and moreover these aims should be for general cultural and not narrowly vocational reasons. Since the article was written, UNESCO has published a booklet *Teaching of Mathematics in Secondary Schools,* and it may come as a shock to many teachers to read therein: "There are no official instructions concerning the aims of teaching the subject at Secondary level. Head teachers of schools are generally free to frame their own schemes of work and to decide such matters as teaching methods, textbooks, educational aims, etc." Since this is the official view of the Ministry of Education in this country concerning the teaching of mathematics, it is little wonder that many teachers may now feel in need of guidance when trying to orientate their teaching methods and syllabuses. The first possible aim for teaching the subject, and perhaps the oldest ever claimed for the subject, is that of a mental discipline; and although it might be felt that this is an objective which has been largely discredited, it may be that some still have a sneaking regard for the claim. For this reason, and for the fact that the exact comments made in the past are helpful to the general discussion on aims, the discipline claim will be dealt with first.

When seeking for support of this claim for mathematics I think we might do worse than consider what Isocrates has to say about it in *Antidosis*. He thoroughly agrees with the comment made by others that mathematics is "nothing but empty talk and hair splitting" because it has no utility nor does the subject contain anything which can be carried over into adult life and used to advantage. Rather its use comes after a knowledge of them has been won. "For in the contemplation we are forced to apply our minds to difficult problems and not let our wits go woolgathering." Mathematics, in the view of Isocrates, was "gymnastics of the mind and a preparation for philosophy." In pursuing these studies for their disciplinary values he was careful to urge his readers not to allow themselves to be misled by these "barren subtleties nor to be stranded on the speculation of the ancient Sophists who maintain that the sum of things is made of infinite elements."

On another occasion Isocrates is equally forthright on the subject of mathematics. In his *Panathenaicus* he begs "those who are inclined to these *disciplines* to work hard, even if the learning can accomplish no other good, at any rate it keeps the young out of many other things which are harmful." He continues by asserting that whilst for the younger generation there is no more helpful or fitting occupation than these studies, for those who have reached man's estate they are unsuitable because mathematicians of his acquaintance signally fail to display the knowledge which they possess, whilst in other activities they are unfitted to be called "educated". He defines this latter state as "those who manage well the circum-

stances which they encounter day by day and who possess a judgment which is accurate in meeting occasions as they arise and rarely miss the expedient course of action."

It is then quite clear that to Isocrates the main benefit gained from a study of mathematics is its disciplinary powers and nothing else. This is not surprising, bearing in mind the function of education in his time, which was mainly centred round the military training of the young men of the ruling classes. After their physical education the next most important feature of their upbringing was the ability to be good citizens by being able to converse about the philosophical mode of the time. This, and the greater chance of face to face relationships which was possible in those times, made it only natural that the art of rhetoric was a highly prized objective in the educational field. The emergence of the theoretical and abstract side of geometry, the work of many mathematicians from Thales, Pythagoras and others, with its development from the simple to the most complex, made it a ready-made educational means for introducing novices to the art of sound argument. With its natural synthesis it was taken as a suitable basis for their further studies despite the fact that the subject matter itself was of no consequence to the educated Greek.

When, therefore, Isocrates is referring to mathematics he undoubtedly has in mind the subject of geometry. In consequence the comments he makes are only valid in respect of that topic and cannot, therefore, provide much help in the terms of the present thesis. Yet to my mind Isocrates is the only one who makes out a good case for the educational discipline of mathematics. For the later writers have either made the claim without supporting it by the slightest suggestion of an argument or by putting forward a case which will not stand up to any inspection. For example, Wormell, Headmaster of the City Foundation School, London, writing in *Teaching and Organisation* (1897) states: "A pursuit of mathematics gives command of attention. A successful study increases or creates the power of concentrating the thoughts on a given subject and of separating mixed and tangled ideas. The habits of mind formed by means of this one set of studies soon extend their influence to other studies and the ordinary pursuits of life. The man or woman who *has been drilled* by means of mathematics is the best able to select from a number of possible lines which may be suggested that which is easiest or most easy to attain a desired end."

It is quite clear that the writer has confused his arguments with those which support a case for mathematics as a training in reasoning powers which was the subsequent factor referred to by him. A second and more glaring error is his attempt to claim, without the slightest grounds, the transfer of training; a matter which when investigated by Thorndike was shown to be with little foundation unless it be to items of similar structure. Again, to suggest that drilling of any description ensures wise selection in non-mathematical situations is a surprising comment from one occupying so eminent a position.

It seems that a good number of those who advocated mathematics as a mental discipline were influenced

(a) by the method used to teach the subject;
(b) by the opposition to the claim for the utility of mathematics.

The method of teaching, right from the earliest times until well into the twentieth century, was one of learning by rote, with frequent drill and heavy punishments for failure; and not until the Greek philosophy on innate ideas was swept away was the claim for the mental disciplining powers of mathematics finally given up and replaced by one based on experience. It is interesting to note that the new philosophy put forward by Locke, Rousseau and others some considerable time before did not succeed in influencing the issue quickly, and it may be that the old discipline claim was kept alive by those who failed to find favour for the utility of mathematics because of lack of agreement as to what constituted usefulness. These last attempts became finally doomed to failure because of similar claims being made with equal force on behalf of other subjects which were coming more and more into the curriculum, and in modern thought the idea of mathematical discipline appears to receive no notice at all.

The final death blow to the claim may well have been the mathematician's desire to have his subject considered as a whole and not as made up of algebra, geometry and arithmetic. The consequent gradual rise in importance of Algebra and Arithmetic and the decline in the stature of Euclidean Geometry throughout the school syllabus therefore made it necessary to search for fresh aims other than those which had been primarily attributed to geometry itself.

Although only two quotations have been given, they are typical of some dozen other writers, and of the two only Isocrates has anything like a case that needs answering. At least, judged by his standards, he can justify his cause even if the child finishes up knowing nothing and remembering little. Unfortunately his defence does not fit into the present-day context, and it would therefore appear that there is not the slightest foundation for any claim today that the study of mathematics by the Secondary School child affords a training in mental discipline.

Our President, who was recently on a mission to Ethiopia on behalf of Unesco, reports the constitution of the Ethiopian Association of Mathematics and Science Teachers, whose purpose is to take the initiative in studying and improving methods, textbooks, conditions of work and the standard of knowledge of its members. Its Chairman is Ato Alemayu (M.A. Chicago) and its Secretary is Woizero Maasa Bekele (B.A. Columbia). We wish our Ethiopian colleagues all the best and god speed in their new endeavours.

A Teaching Centre has also been constituted in Addis Ababa. It is located at the University College of Addis and is sponsored jointly by the College authorities and the above Association. In the centre teachers will find prototypes of aids and notes on their production, a library and the opportunity of attending seminars and conferences.

Ethiopia is a country in full development and needs teachers of mathematics and science as much as any other country, and opportunities for good service and an enjoyable time in a fascinating country can be found there. Anyone interested should contact our President, either directly or through our Secretary.

MATERIALS FOR MODELS—II

J. W. PESKETT

In the previous article the principal material under discussion was cardboard. It is now proposed to consider how metal and small metal articles can be used to make some simple teaching aids.

An inexpensive form of metal rod, which can be obtained in various sizes from $\frac{1}{16}$ inch to $\frac{1}{4}$ inch diameter and about 30 inches long, is mild steel welding rod. The smaller diameters are reasonably rigid and can easily be bent cold into any required shape. The larger sizes will require heating to red heat to give an easy bend for curves of small radius.

Quite neat little models of a variable parallelogram or rhombus with diagonals can be made in the following manner using $\frac{1}{16}$ inch rod. Four equal lengths, say about 5 inches long, are cut and the ends bent, using round-nosed pliers if available, to form small loops of a size to take a brass eyelet. Eyelets are then used to connect the rods in pairs and so form a four-sided figure with hollow pivots at its corners. Diagonals are made from shirring elastic run through the eyelets and along two opposite sides of the figure, the loose ends being tied together to make a continuous loop in tension. As the corners are pivots the parallelogram can be changed to different shapes, but in each one the diagonals are seen to bisect one another. For the rhombus it will be seen that the diagonals always are at right angles. Neither of these two properties will be exhibited in an irregular-sided quadrilateral. If larger models are required, it is advisable to use rod of $\frac{1}{8}$ inch diameter or more, and in this case metal eyelets such as are used for tarpaulins can be used for the "hinges." If one can find a local mechanic who does welding it might be possible to persuade him to try his hand at making cubes, tetrahedra, pyramids, etc., from these rods, so forming useful models for use in solid geometry and trigonometry.

The following will be found of use in connection with three-dimensional problems in trigonometry. Obtain two pieces of perforated zinc, each about 1 foot square, and thread some fine wire loosely through the two pieces along one edge so that one piece is "hinged" to the other. Next, a piece of stiff cardboard or plywood of a size slightly larger all round than the zinc, has a flat beading of cardboard or thin wood, $\frac{1}{2}$ inch wide, fastened all round. One of the pieces of zinc is now fixed on top of the beading with small nails or screws, the beading ensuring that there is a gap between the zinc and the baseboard. It is now possible to turn the other piece of zinc about its "hinge" and when propped up by legs made from welding rod $\frac{1}{8}$ inch diameter, an inclined plane is formed through which it is possible to see. Lines, triangles, etc., may be "drawn" on the inclined plane with chalk, with pieces of welding rod with their ends turned to fit in holes in the zinc, or with coloured elastic bands. The projections of these lines, triangles, etc., on the horizontal plane can now be shown by using $\frac{1}{16}$ inch welding rod as vertical lines inserted through chosen holes in the inclined plane and then through the appropriate holes in the horizontal plane below. The gap mentioned above, between zinc and baseboard, allows these "verticals" to rest securely.

Another source of metal is the material used for curtain rods of the type

that uses brass runners. These rods can be obtained in brass, aluminium, or brass coloured aluminium alloy. With careful bending they may be formed into practically any smooth curve. The "stops," with their screw eye, make convenient "points" which can be moved to different positions. The flat ½ inch rod usually sold in connection with the curtain rods as pelmet fittings, can also be bent to make a curve if required. The small metal fittings for joining two of the pelmet rods together can be cut with a hacksaw into three equal pieces and used as sliding "points". A piece of apparatus for illustrating how a secant PQ becomes a tangent at P when the point Q moves up to the point P, may be made as follows. A convenient length, say 3 feet, of pelmet rod is bent to give a flat shallow curve ½ inch wide. A connecting fitting as mentioned above is cut into three pieces and each part is tried on the curve to ensure a freely sliding fit. To one of these "slides" the male part of a press stud is soldered, while the female part is soldered to another slide. This will allow one slide to move on the curve, the other slide to move on a straight piece of pelmet rod forming the "secant", while the two slides can be joined together by the press stud to give a pivot joint. This can be thought of as the point P. A similar fitting is made for the point Q. If it is desired to fix either P or Q in a chosen position on either the curve or the secant, it is possible to solder a nut to the slide and then a bolt through the nut can be adjusted to press against the rod and keep the slide stationary. (See Fig. 6.) If rubber suction

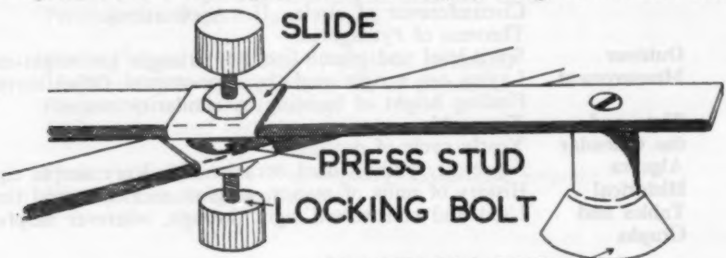


Fig. 6

cups are fastened to the ends of the curved rod the whole apparatus can be supported on any smooth vertical wall or board. Suction cups fixed to other straight pieces can also be used to draw vertical and horizontal lines in any desired position. The cups may be fitted as follows. A small hole is bored through the rod and a small screw put through the hole and screwed into a piece of short dowel of a size which fits tightly into the hole at the back of the cup.

Other sources of metal parts may be Meccano outfits which provide strips, rods, wheels, pulleys, gears, etc. An old alarm clock will provide a number of useful cogs and springs, while shops selling bicycle accessories or shops selling so-called Government surplus stores form happy hunting grounds for all kinds of metal fittings.

(*To be continued*)

First 2 or 2½ years

Arithmetic	Estimation, approximation, applied to measurement, fractions, decimals. Direct proportion. Averages. Inverse proportion. Squares and square roots.
Applied Arithmetic	Household budgetting. Percentage (special percents. only at first); percentage gain and loss. Municipal Finance: Rates. Interest (some work here on decimalisation of money).
Practical Arithmetic	Scales and scale-drawing. Area: Concept, rectangle and right-angled triangle. Irregular areas. Volume: Making cubes and cuboids—volume of these—irregular solids. Representation on paper of 3-D objects.
Geometry	Right-angle and fractions. Set-square triangles. Circle drawing. Parallels. Angle-measuring; angle-sum of triangle and polygons. Height and area of triangle, parallelogram, trapezium. Enlarging and reducing: similarity. Circumference of circle. π. Applications. Theorem of Pythagoras.
Outdoor Measurement	Spirit-level and plumb-line; 45° triangle for height-measuring. Laying out a right-angle by rope-stretch. Offset survey. Finding height of building by similarity methods.
Time and the Calendar	Time-tables. Yearly cycle of daylight.
Algebra	Algebraic symbols used occasionally. Very simple equations.
Historical	History of units of money, weights, measures, and time.
Tables and Graphs	Used and constructed right through, wherever helpful.

SECONDARY MODERN SCHOOL MATHEMATICS

M. J. MEETHAM

The above is a summary of a scheme which attempts to suggest significant topics for Secondary Modern Mathematics, in a reasonable sequence, and with due consideration for the techniques and applications which accompany them. The writer would be glad to enlarge on any of the topics given, but in the limited space of this article will discuss only *Squares and Square Roots, the Use of Logarithms,* and *the Slide Rule.*

Squares and Square Roots

The pupils can discover squares of numbers in their first year. For instance, in revising the multiplication tables the "Multiplication Square" can be built up. (The best and most common way is to write the numbers 1 to 12 along the top and down the left-hand side; and to build each row in turn by filling in the products of the left-hand number and the numbers of the top row.) When complete.

ARY MODERN SCHOOL PUPILS (Upper Streams)

Last 2 or 2½ years

Arithmetic	Use of logarithms for numbers over unity. Slide rule. Logarithms extended to include negative characteristics.
Applied Arithmetic	Compound Interest. Insurance. Income-tax. Municipal Finance; Budgetting. National Finance: The Budget. Trade.
Practical Arithmetic	Prisms (including cylinder). Pyramids (including cone). The sphere. Earth as sphere.
Geometry	Area of circle. Various geometrical curves, plotted as loci.
Trigonometry	Tangent of an angle. Solution of a right-angled triangle. Sine and cosine. Possibly Sine Rule for any triangle.
Outdoor Measurement	Finding height of building by angle of elevation. Map-making by triangulation; use of theodolite. Plane-table method of survey.
Time	Daily and annual variation of sun's altitude.
Algebra	Harder equations (for Trigonometry). Algebraic formulæ.
Historical	History of mathematical topics.

this Square may be used for recognising products (which can be seen also as the number of cells in the rectangle "cornered" by the chosen row and column); and for breaking down a number into two factors. The pupils can then look for numbers that occur *twice* in the Square (*e.g.* 15, 28); numbers that occur *four* times (8, 20, etc.); *six* times (12 and 24 only); *three* times—*once* only—and these latter numbers occur only in the diagonal of the Square and in fact comprise the "squares" of numbers from 1 to 12.

Now make a list of these numbers in the form $1^2=1$, $2^2=4$, etc. Let the pupils continue this list as long as they wish; some of them will notice the pattern by which each square is greater than the preceding one by an odd number, the differences in fact being the consecutive odd numbers from 1. (The explanation of this is clumsy unless algebraic notation is used; as is inevitable for a general statement about number patterns.)

Make a graph of the Squares of Numbers—this will probably be the first

time the pupils have produced a graph which is curved; the scales of the two axes will need to differ, the ratio 1 : 5 is probably the best. The pupils will be able now to use the graph for interpolation: and, if the usual tenth-inch graph paper is used, will consequently gain facility and insight in their handling of decimals.

The teacher can now introduce the concept and symbol of Square Root, and the list of Squares can be "turned inside out" to produce a list of Square Roots. This list of course contains gaps of increasing size; and the pupils can estimate the square roots of the "in-between" numbers such as 5, 12, 21, etc. They can refer again to their Graph of Squares, and use it in reverse for the reading-off of square roots.

Top-stream pupils generally enjoy learning the "long" method for calculation of square roots. Once again, the reasoning behind this process cannot be understood without the techniques of algebra. But the pupils can have the satisfaction of checking by squaring the successive approximations they obtain by the "long" method (*e.g.* 1·4, 1·41, 1·414, for the square root of 2); and this exercise increases their insight into the ideas of approximation and of infinite sequences. Another advantage of the "long" method is that it gives a sure way of obtaining the first significant figure of a square root in difficult cases, when reading from a table.

Commercial tables are of course available for both squares and square roots; and the Squares Table may well be the first table met by the pupils containing mean difference columns—a useful refinement of mathematical tables. Many pupils profit by learning to use this table, particularly if the first one introduced is for *three* figures only. Three-figure tables are very much more concise than four-figure, and quite adequate for Secondary Modern School work; they are unfortunately not often seen in print (the fullest set known to the writer is inside the covers of Durell's "A School Mechanics"), but it is worth while having them duplicated for school use. The Square Root Table, of course, presents an extra difficulty of "choice" of answer which has to be determined by inspection; but all difficulties in table-work can be tempered by suitable selection of examples.

The direct applications of Squares and Square Roots in school work concern *area* problems. In the above scheme, it is suggested that the geometry studied about this time should include further work on areas and the Theorem of Pythagoras. The calculations connected with these topics naturally make use of Squares and Square Roots; but, in the opinion of the writer, the study of Squares and Square Roots thoroughly justifies itself as "Pure Arithmetic."

Logarithms and the Slide Rule

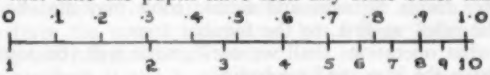
In the writer's opinion, logarithms are best introduced as a useful working tool; and their theory should only be studied by older or specially interested pupils. (This opinion agrees with common practice, but not with the author Burns in his excellent book "Daily Life Mathematics.")

As with the Table of Squares, it is suggested that the first Logarithm Table used should be *three*-figure only. For instance, a good introduction is to present the pupils with a list of the logarithms of integers from 1 to 20, each correct to 3 decimal places. Ask the pupils to calculate log 2 + log 3, log 4 + log 4,

log 20 − log 5, etc.; and so let them discover as a working rule the basic fact about adding and subtracting logarithms. They can now be presented with three-figure logarithm tables, and readily discover how to use these for calculations involving products, quotients, evolution and involution.

All *numbers* used at first should lie between 1 and 10, so that the characteristic of every *logarithm* is zero. But the extension to use of logarithms of numbers over 10 is an easy step, now that the product rule is understood: for $\log 2 = 0{\cdot}301$, $\log 10 = 1$; so $\log 20 = 1{\cdot}301$ and $\log 200 = 2{\cdot}301$, etc. The extension to include negative characteristics is more difficult, and can be deferred for some time; once this is mastered, however, logarithms can be used for trigonometrical calculations.

To make and use a simple Slide Rule is interesting and valuable, and need not wait until negative characteristics have been studied. The teacher can draw a logarithmic scale using the logarithms of the integers from 1 to 10—this will probably be the first time the pupils have seen any scale other than a plain scale.



The diagram shows plain scale graduations along the top, and corresponding logarithmic graduations underneath—the plain scale can later be erased. The teacher will also need to show how two congruent rulers can be used together to perform addition and subtraction of numbers. The pupils can now each make a logarithmic scale, split it into two congruent scales, and operate these together as a "slide rule". Linear-logarithmic graph paper can now be issued, and the pupils can use it to make more accurate slide rules. Commercial slide rules of all kinds are also of great interest to some pupils; and, although calculating machines have superseded slide rules in many departments, yet the pupils should realise that slide rules remain indispensable for individual use in many occupations.

Dr. J. C. Evans of the National Physical Laboratory, writing in the December issue of *Discovery* on *Gravity and the Olympic Games* discussed the effects which the different values of g at Helsinki and Melbourne might be expected to have on athletes' performances in the field events. This difference is 1 part in 500, which is sufficient to produce an increased range at Melbourne in the throwing events of about 0·2%. Records are often broken by considerably less than this. On the other hand the corresponding differences in the jumping events are hardly big enough to be of practical importance. They amount to 1/10 inch, $\frac{1}{2}$ inch and $\frac{1}{3}$ inch in the High Jump, Long Jump and Pole Vault respectively.

Dr. Evans was writing before the Games took place, but now that the results are known we can see that many record breakers may claim full credit for their achievements. In the men's events P. O'Brien (United States) put the weight 60′ 11″, A. Oerter (United States) threw the discus 184′ $10\frac{1}{2}$″, and E. Danielsen (Norway) threw the javelin 281′ $2\frac{1}{4}$″. These are improvements of 3′ $9\frac{1}{2}$″, 4′ 4″ and 3′ $1\frac{1}{2}$″ on the old records, and similar increases were achieved in the women's events.

OPERATIONAL ALGEBRA

R. M. Fyfe

The simplicity of genius lies behind this amazingly successful method of beginning Algebra, which has now been tested thoroughly in several English schools. Dr. Gattegno's outline description of his method is contained in *L'Enseignement des Mathématiques*, Geneva, 1955. My use of this information has enabled me to work out a first year's course, and in this article I am explaining what can be done in the opening stages.

I invite the class to join in a "game with numbers." The rules of the game have to be determined and I ask what they think I mean by "operations". They find that I mean things we *do* in arithmetic, and they can soon name: add, subtract, multiply, divide. Squares and square roots may also be mentioned. We agree to limit the rules, at first, to the familiar four.

"If I say 'what operations shall we use?', what will you say?"

They now have the idea, and we begin.

"You, Anne, think of a number from 1 to 10, but do not tell us—it is your secret." "You, Basil, tell us a number." He says "8."

"You, Carol, what operation shall we use?" She says "multiply."

"Anne—you must tell us the answer. You have to multiply your number by 8." She says "56."

"Now, we all have to guess her number. Hands up when you know."

Hands shoot up, and when all are ready, I ask many children. They all say 7. "You *all* say 7. Is it right, Anne?"

We go on, using any operation they choose. Everyone can do it. At the end of the lesson we discover what it is we have been doing, and I elicit the pairs: multiply—divide, divide—multiply; add—subtract, subtract—add. I give the word "inverse" and write on the blackboard (which I have not used at all in this lesson) *INVERSE OPERATIONS*. I also *say*, "to *undo* multiplication, we divide," etc.

In the second lesson we decide that it was too easy, and that we will use two operations, each time. At first, with add, subtract and multiply as the choice.

There are a few pupils who find some difficulty in remembering all that the others have said in the making up of the "sum" (I do not call them equations, yet). To help them it is agreed that I shall write it on the board, step by step. The secret number is represented by the child's initial. Strict order is maintained, and all signs are put in. Everyone understands what I write. The children can soon explain that the *last* operation is undone before the first. They have to tell me what to write for this middle stage. For example:

$$P \times 7 + 3 = 31$$
$$P \times 7 = 28$$
$$P = 4$$

The opportunity is bound to arise for having to make clear that add 3 to P and multiply by 7 may be ambiguous, when written, as it could be confused with multiply P by 7 and add 3. They agree to understand the first by $(P+3)\times 7$, and the second by $P\times 7+3$.

We go on, orally, and I continue to write on the blackboard, until everyone is getting all the sums right. I always invite the holder of the secret to confirm the findings of the others.

It is now time for each individual to begin to make his own examples. This they do, and write them on the blackboard, and everyone sets to work on the examples of the others. They criticise each other's errors, when these occur. They copy down the rest, and do them for homework. No one ever forgets to do this homework, or says it is too difficult, although in fact the equations are much more complicated than those of the textbooks.

Numbers can be extended to 20 if it seems desirable.

In the third lesson, we include division in the operations to choose from. The written sign for it is to underline what we had so far, and put the new number as a denominator. Difficulties now arise, as fractions begin to crop up as results. One way out of the dilemma would be to go back and revise fractions, but I suggest as an alternative that *I* should invent the secret numbers for them. All can now perform the inverse operations, but I feign fatigue at having to do so much thinking, and suggest a new course.

"Why have numbers at all? It will be *much* easier if I just use your names."

The first example, from John and Betty and Pamela gives me $\frac{J + B}{P} = F$ (I take responsibility for the result, as I was doing just before.) All the class heard the order in which the operations were said, and saw me write it, step by step. So they know that they first have to multiply by P: $J + B = F \times P$ and then subtract B, and there is John, clear again: $J = F \times P - B$ is the answer.

All agree that this is much easier. After a few more examples, orally, they against set to work to make up an exercise for everyone else to do. Someone is sure to want to put in more than two operations. I am willing, if they will show me the answer before they write it on the blackboard. It goes into a different set, labelled "harder."

The intense liveliness and interest in these lessons, and the complete absence of any difficulty about notation, has to be seen to be believed. But every teacher can discover this for himself, and I hope many will do so. I shall be very interested to be told what happens.

Henri Pousseur and Karlheinz Stockhausen have both stepped off from the resources of electronic music, in which tones can be measured, and they apparently measure the ingredients of their music, the pitches and names of notes, the morphology of touch and phrasing, and so on. Stockhausen has a disciple called Bo Nilsson who lives in the Arctic Circle and has learnt modern musical techniques by Correspondence course, it is said; he constructs his music by the theory of groups.

Earl Brown composes according to a system of geometrical theories, the theory of numbers, and an assortment of graphs which the performer imposes one upon another at his own discretion; his pieces are actually called Systems.

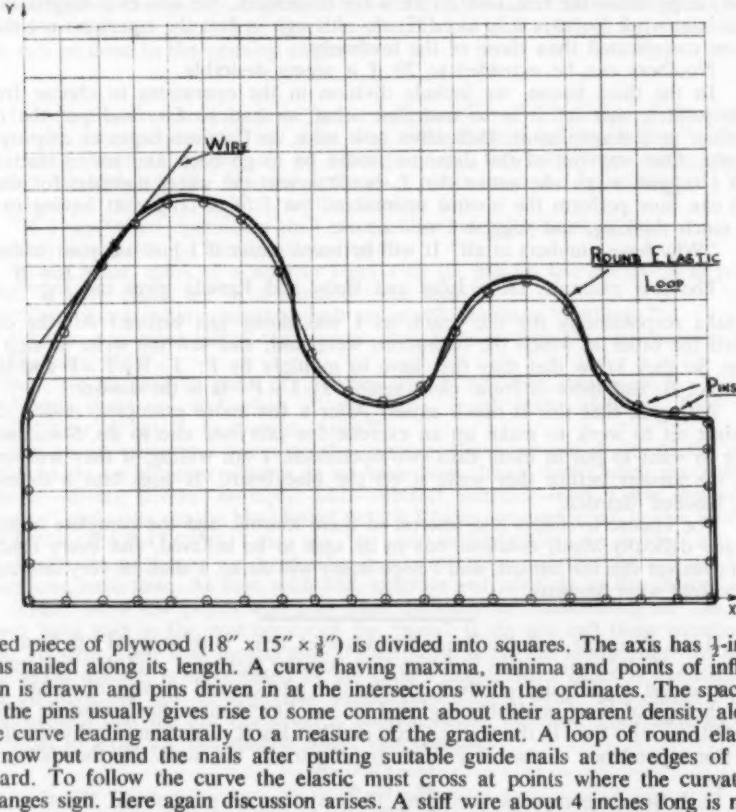
—From a report in *The Times* on a recital of piano music played by Mr. David Tudor at the International Music Association Club.

SOME APPARATUS FOR THE TEACHING OF CALCULUS

C. HOPE

1. Differentiation

In discussing the relationships between the gradient, tangent and shape of a curve the device illustrated in fig. 1 has been found most stimulating. A suitably



sized piece of plywood (18″ × 15″ × $\frac{3}{8}$″) is divided into squares. The axis has $\frac{1}{2}$-inch pins nailed along its length. A curve having maxima, minima and points of inflection is drawn and pins driven in at the intersections with the ordinates. The spacing of the pins usually gives rise to some comment about their apparent density along the curve leading naturally to a measure of the gradient. A loop of round elastic is now put round the nails after putting suitable guide nails at the edges of the board. To follow the curve the elastic must cross at points where the curvature changes sign. Here again discussion arises. A stiff wire about 4 inches long is now attached to the elastic by a binding of cotton. By pulling the elastic along the x-axis the tangent is made to traverse the length of the curve. It is usually necessary to 'ease' the pins over to allow for a smooth passage. Changes of slope are clearly demonstrated and stationary points are evident. This device has the advantage that

it is available on subsequent occasions for reminding or revising and the movement of the tangent requires no act of imagination.

2. Integration

The integral as the inverse of the gradient is usually given no geometrical demonstration. The following devices may be of interest. I believe, although I have no reference, that the idea is due to Arndt in the latter half of the last century.

Given a curve, draw the ordinate (length $f(a)$, say) at $x=a$, join its intersection with the curve to the point $(a-1, 0)$. This line has gradient $f(a)$ and is therefore parallel to the tangent to the curve which is the integral of $f(x)$ at the point where $x=a$. Using a parallel ruler, the integrated curve may be drawn as the envelope of a succession of tangents (fig. 2). The constant of integration fixes the position of the curve relative to the existing axes.

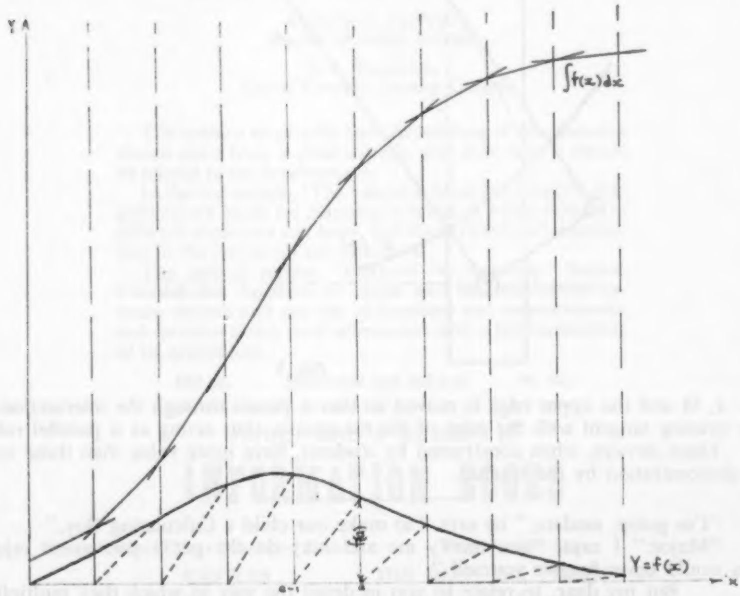


Fig. 3 shows a device made from Meccano and thick card which facilitates the drawing of the integral. A tee-square is made from a piece of thick cardboard 18″ long, $2\frac{1}{2}$″ wide. A parallelogram from jointed Meccano rods is fixed to a point 1 unit from the leading edge. The lower side of the parallelogram joins $(a, f(a))$ to

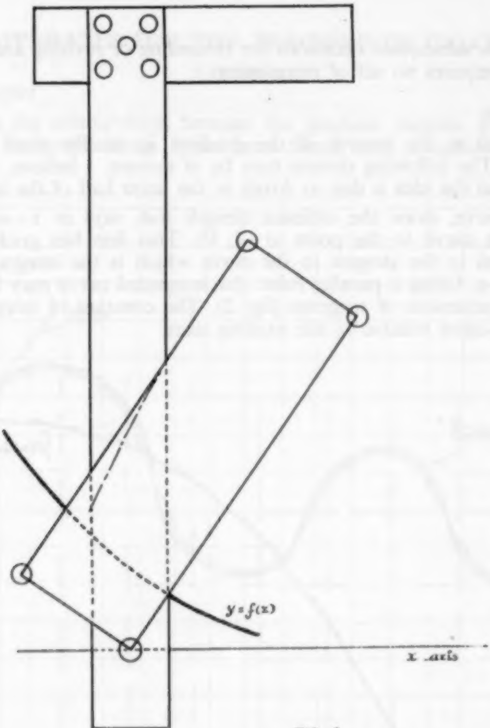


FIG. 3

$(a-1, 0)$ and the upper edge is moved so that it passes through the intersection of the existing tangent with the edge of the tee-square, thus acting as a parallel ruler.

These devices, when constructed by students, have more value than those used in demonstration by the teacher.

"I'm going, madam," he says, "to make our child a Calculating Boy."

"Major," I says, "you terrify me and may do the pet a permanent injury you would never forgive yourself."

But my dear, to relate to you in detail the way in which they multiplied fourteen sticks of firewood by two bits of ginger and a larding-needle, or divided pretty well everything else there was on the table by the heater of the Italian iron and a chamber candlestick, and got a lemon over, would make my head spin round and round and round as it did at the time.

CHARLES DICKENS, *Mrs. Lirriper's Lodgings.*

POTTLES, COOMS AND WEYS

A periodical overhaul of the bookshelves brings forgotten treasures back to light. So it came about that I rediscovered my copy of Walkingame's *The Tutor's Assistant: Being a Compendium of Practical Arithmetic*; first published in 1751 the work ran to many editions and the use of the book became established in "almost every school of eminence throughout the kingdom." My own copy is marked "One hundred and seventy ninth edition" and is published by the firm of Hammond, Eight Brothers. It bears no date and may be a 'pirate' edition, but whatever its history it shows vividly the mathematics which was taught two hundred years ago. Numbers stay the same, but fashions change.

The book starts with elementary questions on numeration. "Numeration teacheth the different value of figures by their different places, and to read or write any sum or number." Every term is given a formal definition in this style. "Multiplication teacheth how to increase the greater of two numbers given, as often as there are units in the less; and compendiously perform the office of many additions." "Barter is the exchanging one commodity for another, and informs the traders so to proportion their goods, that neither may sustain loss."

How many teachers today know the reason for the name "Practice" as applied to a particular way of multiplying money, or how many can explain the mysteries of Alligation? It is all here. We learn that alligation is either *medial* or *alternate*, that the Rule of False is divided into two parts, *Single* and *Double*, and "is a rule that, by false or supposed numbers taken at pleasure, discovers the true one required." The Double Rule of Three is explained in the following terms. It "is so called, because it is composed of five numbers given to find a sixth; which if the proportion is Direct, must bear such proportion to the fourth and fifth, as the third bears to the first and second. But if Inverse, the sixth must bear such proportion to the fourth and fifth, as the first bears to the second and third. The three first terms are a *supposition*, the two last a *demand*." After this one understands the old jingle:

"The Rule of Three
Puzzles me!"

There are pages of examples on the extraction of the square, the cube and finally the *n*th root; but logarithms are not mentioned, and the world record for π is Van Culen's thirty-six places which are "engraven upon his tombstone in St. Peter's Church in Leyden." The final chapter is entitled "Promiscuous Questions" and one reads on avidly; but alas! it is only what today would be called "Miscellaneous Examples."

The problems take us back to a vanished age. "An army consisting of 20,000 men, took and plundered a city of £12,000. What was each man's share, the whole being equally divided among them?" Gentlemen regularly send tankards to the goldsmith for minor repairs, they wear cloaks of plush, cloth is bartered for beeswax and a lady's dowry includes a petticoat with two rows of furbelows. Vintners mingle wine (to provide examples on alligation), families are of a phenomenal size and a

young scholar "coming into town for the convenience of a good library" can get board for £10 a year. Some of Merrie England still remains for our old friend the snail (who climbs up eight feet during the day but slips back four feet every night) is still climbing and slipping on a May-pole.

But the great glory of the book is its units and its bills which show us a land of ells of diaper and dowlas, fine laced tippets, Balladine silk, yards of drugget, shalloon and drab cloth. There are firkins of butter at 3d. a pound, but fine hyson tea is 23s. 6d. We read of fothers of lead and puncheons of prunes, of nutmegs and tabby and bohea. The maze of units testifies to the sturdy independence of the English merchant. We find the coon, the strike, the pottle and the last; the clove, the scruple, the hogshead and the nail; the firkin is eight gallons in London, but in other parts of England, with greater liberality, it is eight and a half. Every possible ratio occurs in a world which had never even heard of the metric system. There are threes and fives and sevens, and the "five and a half yards one rod, pole or perch" of our childhood is accompanied by six and a half tods to the wey.

But if England has changed the section on foreign currency shows that our neighbours have changed still more. "In Paris they exchange by the crown; the exchange in Florence is by ducatoons; they keep their accounts at Madrid, Cadiz and Seville in dollars, rials and marvedies." All these foreign currencies are past memories, the stiver and the groat are gone but the pound sterling still remains.

Arithmetic in those days was clearly taught as a conglomeration of loosely connected facts to be memorised, and rules to be applied—old-fashioned methods which we like to think are superseded. But in his day Walkingame was a pioneer; the idea of using a book at all when teaching was by no means firmly established, and in his preface he is at great pains to defend himself against the objection that to teach by a printed book is an argument of ignorance and incapacity. If today Walkingame's Arithmetic is interesting only as a book on social history how many of our present texts will shortly be the same? Are we truly up to date, and are our methods changing as they should to meet the new needs which are arising? How will high-speed computation affect requirements in elementary teaching? When automation is a frequent topic of conversation what are we to teach the boy who will design the electronic office or manage the automatic factory? With the coming of the high-speed computor is not much of our own teaching already as dead as the pottle, the coom and the wey?

T.J.F.

A Belgian has invented a system of teaching children the basic mathematical operations—addition, subtraction, multiplication and division—through the physical handling of coloured rods of varying lenghts.

Many think this an unnecessary complication of an earlier and simpler system of teaching children the basic mathematical operations through the physical handling of one plain rod of uniform length.

PETER SIMPLE in *The Daily Telegraph*.

A.T.A.M. FILM UNIT

It is now between two and three years since the proposal that the Association should produce its own mathematical films was first made, and it may therefore seem to some that progress has been slow, but work of this kind is not being done by any other group of mathematics teachers anywhere else in the world and before the first film can be shot a great deal of preparatory work is needed. As we announced earlier a generous grant from the Nuffield Foundation has enabled us to buy a 16mm. camera and to have it modified for animation work. Mr. I. Harris has designed a rostrum on which the camera is to be mounted and the work building this rostrum is now well advanced, in fact we hope to have completed the first tests on it by the time this report appears in print. Drawings on a number of different topics have already been prepared and experimental shooting will commence as soon as the rostrum and camera are installed.

A number of people have expressed interest in this project and willingness to help with the production work. The main difficulty is to combine volunteers who live in widely different places into an effective team, but we hope that some way of doing this will be found. Meanwhile can we ask those who so kindly offered us assistance to keep in touch with us and to await further announcements in these columns.

The constitution of the Film Unit was agreed some time ago by the Association Committee but it has not been printed before so it is given below and it will make the aims and structure of the unit clear.

CONSTITUTION

The production unit shall work within the framework of the A.T.A.M. under a Director appointed by and responsible to the Main Committee of the Association. The purpose of the Unit is to encourage the study and use of mathematical films, and to undertake the production of such films either on their own or in conjunction with other organisations.

The day to day administration of the Unit shall be the responsibility of the Director. He shall inform the Main Committee of commitments which are undertaken in the name of the Association, and he shall notify the Committee of any films which it is proposed to circulate bearing the Association's name.

The organisation of the Unit at any particular time will depend upon the production programme in hand. Announcements about the work will be made periodically in the Association's Bulletin, and the Director will endeavour to find suitable employment for any member of the Association who wishes to join in the work. At the same time it will obviously not always be possible to do this, and the first task of the Production Unit is the production of films, and the training of members of the Association in the techniques involved is merely incidental to this.

Any rights in the films produced by the Unit will become the property of the Association, and the Director will administer them on the Association's behalf.

The list of Mathematical Films and Filmstrips suitable for use in Schools included in the last number of this journal has been printed separately as an eight-page pamphlet. Copies may be obtained price 6d. (plus postage) from the Secretary.

A.T.A.M. FILM UNIT

STATEMENT FROM FOUNDATION UNTIL 12TH JANUARY, 1957

Income	£	s.	d.	Expenditure	£	s.	d.
27.6.56—From Nuffield Foundation	240	0	0	19.7.56—B. J. Lynes, for modifying camera ...	200	0	0
From Mathematical Pie, Advance on joint production of film ...	50	0	0	27.7.56—Stamps on cheques ...		4	0
				10.9.56—To T. J. Fletcher, for items listed on separate account	56	2	3
				12.1.57—Balance in hand ...	33	13	9
	£290	0	0		£290	0	0

Audited and found correct.

J. W. PESKETT. T. J. FLETCHER (Director).

23rd January, 1957.

ABERDEEN

A Conference on the Teaching of Arithmetic and Mathematics was held in the High School for Girls, Aberdeen, on Saturday, 13th October, 1956. The programme was as follows: —

Primary. 10.30-11.15. Demonstration Lesson, Infants (aged 6), by Dr. C. Gattegno, Institute of Education, University of London.

11.30-12.15. Demonstration Lesson, Juniors (aged 9), by C. Hope, Esq., City of Worcester Training College.

2.00-4.00. Talks on *Mathematics in the Primary School,* by Dr. Gattegno and Mr. Hope, followed by discussion.

Secondary. 11.30-12.15. Demonstration lesson with film-strip, *Third Year Senior Secondary,* by R. H. Collins, Esq., Technical High School for Boys, Doncaster.

2.00-2.45. *Non-Examination Mathematics in the Secondary School,* by Miss Y. Giuseppi, Dick Shepherd Comprehensive School, London.

3.00-3.45. *The Significance of Mathematical Films in Education,* by I. Harris, Esq., Dartford Grammar School, Kent.

Exhibition. 10.00-4.00. Mathematics Models, Films, and Film-strips, Mechanics Apparatus for Junior Secondary Pupils (lent by Mr. A. Arkley, Hilton Secondary School, Aberdeen).

This Conference was a great success, not only in numbers—about 280, one-third of them secondary, teachers took part—but also in the amount of discussion it promoted in the meetings and in staff-rooms the following week, for some of the ideas presented were new enough to provoke both antagonism and enthusiastic approval. One of the most notable features of the Conference was the teamwork of the demonstrators and lecturers.

It is very likely that the Cuisinaire apparatus will be experimented with in some Infant Schools, and many Junior teachers are thinking about the possibility of using it for backward pupils at later stages. It was unfortunate that some Primary teachers were unable to return to the afternoon discussion, as those who did found that many of the problems arising out of the morning demonstrations were resolved later. The suggestion has been made that it would have been better to have the discussion first and the demonstration second. This is more a criticism of the organisers than of the demonstrators, but the provision of classes on Saturday afternoon offered some practical difficulties.

Criticism of the junior lesson was governed by the fact that only arithmetic is taught in our primary schools in Scotland. Teachers felt that although they admired the simple ingenuity of the geometry boards, and the ease with which the class was handled, the subject matter was rather too far removed from their day to day requirements. In fact, in Aberdeen, the boards would appear to have more of a future in the lower streams of Junior Secondary Schools than in the Primary. At the same time, as one younger member of the Conference remarked, "Isn't it refreshing to hear someone suggest, for a change, that children should be asked to do more rather than less than they are now doing?"

Notes received by the Secretary to the Conference from primary teachers were peppered with remarks like "interesting", "stimulating", "thought-provoking".

In the secondary section, Mr. Collins's lesson demonstrated, not only his own

ability and enthusiasm as a teacher, but drove home the value of the film-strip as a medium of teaching. The rapidity with which new figures could, as it were, be drawn on the board, with the possibility of returning to a previous diagram in a matter of seconds, illustrated the saving of time in the subject of graphs, which can by ordinary blackboard methods be so time wasting. In fact, the general tendency here is to neglect the subject. The value of film-strips in rapid revision might also have been mentioned. This no doubt might have emerged in discussion if lunch-time had not been so near. Few senior teachers ever see their subject taught and it was refreshing to see "oursels as ithers see us", except for the fact that we would have been much more self-conscious.

Miss Giuseppi's talk was interesting and helpful in that she concentrated on attempted solutions rather than on the mere statement of problems. One Mathematics teacher from a small comprehensive country school writes: "We are faced with the problem of conflicting lines of attack to our various streams, and the Conference showed us that we have hardly touched the fringe of the problem." At the same time the talk did rather emphasise the common experience here, that it seems to be easier to find a line of attack for lowest stream girls than for lowest stream boys, but it would be unfair to criticise Miss Giuseppi for discussing the side of the work she knew at first hand. It does, however, occur sometimes to the senior secondary teacher to ask if it is really worth while to try to teach mathematics so far down the intelligence scale.

Mr. Harris, like other members of the team, stimulated by his own obvious knowledge of, and keenness for, his subject. As one teacher put it, "If it made hardened teachers sit up, so it would pupils." There is no doubt that the film is *the* method for illustrating loci and envelopes. There seems to be little hope of using such an aid in this area until we are much better equipped with apparatus which can be used casually as part of the lesson and not just used as a special effort once in a while in a special classroom. Mr. Harris's suggestion of letting ideas grow from the film instead of teaching a preconceived lesson from a figure drawn on the blackboard, might be countered by the thought that the film as well as the blackboard figure is chosen by the teacher, and that the film has been constructed with certain ends in view.

The exhibition demonstrated the extent to which we lack material aids in Scotland. There is, perhaps, not much need for these aids at the top end, as surely mathematics for mathematicians is an intellectual process, and good pupils should be encouraged to use their imaginations as early as possibie. We tend, however, still to think too much in terms of a watered-down academic course for non-mathematicians. Only the well-qualified mathematic teacher is sufficiently informed to make the practical approach useful and interesting and too often his own training and inclination pull him the other way. Mr. Arkley's exhibition demonstrated the value of big models and the advantage that comes from trimming off all mathematical complications so as to leave principles as clear as possible. It is a pity that his work is not more widely known.

One might sum up in the words of a secondary teacher, "It is always a good thing when those interested get together to *think* about teaching mathematics." From that point of view the Conference was an outstanding success. D. DONALD.

ANNUAL GENERAL MEETING

The 4th Annual General Meeting of the Association was held on Saturday, 2nd February, 1957, at the Institute of Education, Malet Street, London, W.C.1.

Mr. R. H. Collins took the chair, and explained that Dr. Gattegno was in Ethiopia on an UNESCO mission. Twenty-six members were present.

Mrs. Fyfe reported that she had been acting as Secretary since September, and thought this meeting was the appropriate opportunity at which to express the Association's gratitude to Miss Giuseppi for her hard work as Secretary over many years.

Successful meetings had been held in London in May, and in Aberdeen in October, 1956, and plans for 1957 included today's open meeting, day conferences at Nottingham in March, and at Leicester in October, and week-end courses in Blackpool in March and near London in November.

No. 2 and No. 3 of the bulletin had been published in 1956.

Miss Briggs presented the Treasurer's report, and the accounts showed a balance in hand of £46 5s. 2d.

Alterations to the Constitution were explained and adopted.

The members of the Committee elected were: R. H. Collins, Chairman; Mrs. R. M. Fyfe, Secretary; Miss B. I. Briggs, Treasurer; and Miss J. Clarkson, Miss Y. Giuseppi, Messrs. C. Birtwhistle, T. J. Fletcher, R. H. Fielding, C. Hope, I. Harris, R. D. Knight, J. V. Trivett.

The Chairman then expressed the Association's deep debt and gratitude to Dr. Gattegno, and explained that the alterations in the Constitution made it possible for him to be elected as the first President.

Questions and discussion on the work of the Association were invited.

The Secretary introduced a campaign to double the membership during 1957, and asked everyone to try to get new members. R. M. F.

The Open Meeting of the Association on 2nd February, 1957 was held at the Institute of Education, Malet Street, London. It took the form of an exhibition of models, films and filmstrips at which 'workshop talks' were given on the making and use of the aids on view. More than 150 people attended, and great enthusiasm and interest were apparent, nearly fifty new members being enrolled.

Messrs. Peskett, Swinden, Fielding and Collins spoke in turn on various aspects of their work and it was a particular pleasure to note so many new models and pieces of apparatus being demonstrated for the first time. Considerable interest was shown in the Cuisenaire rods and Gattegno Geo-boards, which were explained to individuals and to small groups throughout the day by Mrs. R. M. Fyfe and Mr. J. Trivett.

In a separate room Mr. I. Harris showed films and filmstrips and in the afternoon Mr. T. J. Fletcher lectured on the Design of Mathematical Films,

illustrating his talk with further examples. This part of the exhibition showed not only finished films and filmstrips but also displayed some of the techniques of producing them and some experimental stereoscopic slides in colour.

It had been the intention of the Committee to vary the pattern of the exhibition and adopt a slightly different plan from our earlier displays, and with this aim in mind there were fewer formal lectures and the emphasis was placed on the workshop talks. This resulted in both gains and losses. The unexpectedly large numbers attending tended to overwhelm some of the informal demonstrations, and as a result some of the visitors perhaps could not gain information which they were seeking; but on the other hand it was possible for the demonstrators to meet a few real enthusiasts individually and discuss important details of the work with them. It is from members such as these that new ideas will come in the future, and it must be the determined policy of the Association to help and encourage them. The aim at future demonstrations must be to combine a general display to the public at large with opportunities for individual discussion.

ASSOCIATION FOR TEACHING AIDS IN MATHEMATICS

1st January, 1956—31st December, 1956

INCOME

	£	s.	d.
By Balance Sheet 1st Jan., 1955	94	11	7½
„ Subscriptions	102	15	7
„ Sale of Films	10	0	0
„ Gift towards Bulletin ...	57	0	0
„ Sale of Bulletin	1	7	0
„ Advertisements	4	0	0
„ Refund Travelling Expenses		10	0
	£270	4	2½

EXPENDITURE

	£	s.	d.
To Cost of printing Bulletin I ...	37	19	7
„ Cost of printing Bulletin II ...	73	2	0
„ Expenses—Editor	7	12	5
„ Review Editor	1	0	0
„ Cost of distributing Bulletin III	4	0	0
„ Expenses for Bristol Week-end Exhibition	7	2	0
„ College of Preceptors (Film Day)	7	11	6
„ Fares for children to attend Film Day	1	19	7
„ French Institute (Films) ...		18	0
„ A.T.A.M. Film Unit ...	50	0	0
„ Purchase of Films	6	13	4
„ Printing of Membership Forms	6	7	0
„ Cost of half return fares for members to attend Committee Meetings	5	8	10
„ Secretary (Petty Cash) ...	7	11	6
„ Treasurer (Petty Cash) ...	5	13	11½
„ Bank Charges, Revenue stamps on cheques ...		19	4
„ Excess of Income over Expenditure	46	5	2
	£270	4	2½

B. I. Briggs, Hon. Treasurer (signed) 3.12.56.
Audited and found correct. J. W. Peskett (signed) 2.2.57.

IMPRESSIONS FROM NOTTINGHAM

The success of the one-day course, held at Nottingham on March 23rd, is something of which the A.T.A.M. may justifiably feel proud. A well organised and interesting programme and the pleasant surroundings of the University on a spring day combined to make the course very worth while.

The exhibition of teaching aids attracted considerable attention. It was arranged, with mathematical precision, so that one could easily find whatever may have been to one's own preference and, at the same time, be attracted by something new or different. Was it by accident or design that the darkened end of the room, arranged for demonstrating films and filmstrips, became a comfortable resting place for a number of weary mathematicians?

The subject of Mr. Trivett's lecture was one with a wide appeal. The slower forms in the Grammar schools and the quicker forms in the Secondary Modern schools must inevitably bear a close resemblance, and Mr. Trivett combined the problems arising from teaching these forms in such a way as to dissolve the barrier, which exists in the minds of many people, between the Grammar school and the Secondary Modern school. The problems were presented and discussed in an interesting but amusing way which stimulated much discussion during a most enjoyable lunch. However, the conversations during lunch were not entirely mathematical; the modern decor of the dining room supplied a fairly general topic of interest—perhaps it did have a certain mathematical appeal!

The A.T.A.M. organisers have developed an ability to make those who attend a one-day course feel that they are a part of it, and I am sure that many of the teachers who went to Nottingham were glad of the opportunity to exchange views as well as to hear the arranged lectures. The overall effect was to bring a sense of proportion to the problems arising from teaching, under conditions which are not always ideal, what is now regarded as one of the most important subjects in any school.

I.L.C.

MATHEMATICS IN THE SECONDARY SCHOOL

For the first time in the history of the Association sea air, a luxury hotel and honeymoon couples comprised the background of a week-end course. The course took place at Redman's Park House Hotel, Blackpool, from March 29th to 31st.

Friday evening was spent on the question "What should be our aim in teaching mathematics?" The discussion was introduced by Mr. C. Birtwistle (Nelson Secondary Technical School). Vocational reasons, passing examination, a discipline, a study for its own sake, utilitarian purposes and a language for science were all suggested. It soon became clear that the wide range of ideas as to what mathematics was, and the wide range at which those present were teaching it would make progress slow, but fairly general agreement was eventually reached that the aim of education was the development of the child's personality to its fullest extent, and the role of mathematics in this was left as a question for homework.

Next morning Mr. C. Hope (Worcester City Training College) spoke on "The position of aids in modern methods". Illustrating his talk with numerous aids he showed how he could teach for *insight*—an understanding—and for *reflex*—a skill. He made the final point that the teacher should be convinced by the child rather than the child by the teacher. Anyway, Mr. Hope, the teacher, convinced us! Two lessons followed. Mr. R. H. Collins (Doncaster Technical School) using a filmstrip, Introduction to Graphs, with 12 year-olds, showed how much could be achieved even under demonstration conditions. Amongst the information obtained was the surprising fact, at least to the class, that our Chairman's age was thirty-six. Film and filmstrip featured in the lesson which Mr. T. J. Fletcher (Sir John Cass College, London) gave to a group co-opted from members of the course. We rocked and rolled our way to find how to draw tangents and normals to epi- and hypocycloids. It occurred to many to wonder why parts of this lesson were carried out in total darkness. Was this a new use of a visual aid?

Before being set free for the evening Mr. Fletcher then opened a discussion on "How methods fail". Among the points which he put forward he said that human relations were more important than methods. There are teaching aids and gimmicks, the aid mirrors in its structure the structure of the mathematics presented while the gimmick only has a mnemonic value. He made a plea for inductive rather than exclusively deductive methods and for the development of geometrical as well as algebraic argument.

Throughout the day we were able to browse over an exhibition of teaching aids and a display of teaching books; the latter kindly arranged by the Lancashire County Library.

On Sunday we were at work again without even the luxury of a late breakfast. Mr. R. H. Fielding (Mountgrace Comprehensive School, Middlesex) spoke on "Lines of future development in the teaching of mathematics" and gave us the opportunity to peep behind the scenes into the methods and content of the mathematics taught in a comprehensive school. The aim of imparting a love of mathematics was uppermost with practical and dynamic geometry taking a vital part. The importance of school mathematics societies was stressed. The audience then split up into smaller groups for discussion, but time was too short for many conclusions to be reached.

At the concluding session on the Sunday afternoon it was unanimously agreed that we should have another conference there in 1958, perhaps of a more practical nature with discussions on the making of teaching material and more demonstrations. In addition a Lancashire group was formed within the Association and it was decided to hold the first meeting between Easter and Whitsun. Mr. Birtwistle was to act as convenor and it was thought possible that the meeting place could alternate between Preston and Manchester. This alone will testify to the success of the week-end, and once more the Association owes its thanks to Mr. Birtwistle and those who helped him for the excellent organisation and the hard work of preparation.

I.H.

Barbirolli's orchestra matches Williams' enthusiasm note for note, dyne for erg.

Magazine TIME.

COMPUTER IN A TECHNICAL COLLEGE

A Dekatron computer, which was at the Atomic Research Establishment at Harwell, has been purchased with industrial funds by Oxford University Mathematical Institute and Delegacy for Extra-Mural Studies and put up as a prize in a competition for technical colleges. The computer has been awarded to the Wolverhampton and Staffordshire College of Technology. Mr. R. Wooldridge, Senior Lecturer in Mathematics at the college, writes: —

The use of a computer by schools is a venture in which success will not be easy. A number of local schools have expressed their willingness and enthusiasm to take part in the experiment. For the past four years the College has been arranging Christmas lectures on scientific subjects for schools, and it is intended to introduce the computer to schools via a series of "popular" lectures of this nature on the history of calculating instruments and machines and the use of such machines in the fields of industry, science, engineering and commerce. We will use as much demonstration equipment as we can construct or obtain, any films that are available on such machines, and representatives from local industry. In addition we are going to arrange courses for sixth form mathematics and science pupils on elementary Numerical Analysis so as to introduce pupils to methods of computation, the problem of programming, and the possibilities of a career in industry using these machines. The approach in such a course would be an "experimental" one so as to encourage the students to discover problems suitable for solution by the computer and so give them the opportunity of programming and carrying out calculations on the machine. We also intend to make the computer available to schools for such work as the construction of any mathematical tables required by local schools.

FORTHCOMING EVENTS

May 31st, 5.30 p.m. at Institute of Education, Malet Street, W.C.1.
Forum of Teachers in Modern Schools.
Non-members also welcome.

June 1st, 11 a.m. to 4 p.m. Cambridge Institute of Education.
The Primary School.
Those who want to attend should apply direct to the Organising Tutor, 2 Brookside, Cambridge.

October 12th, Day Conference in Leicester in conjunction with L.E.A. Details later.

November 8th—11th, Residential weekend near London.
The Modern School.
Only a limited number will be accepted and provisional application should be made at once to the Secretary.

The following periodicals have been received: The Australian Mathematics Teacher, Vol. 12, Nos. 1, 2 and 3; Ensenanza Media, No. 3.

Miss I. L. Compbell, Mr. J. W. Peskett and Mr. L. A. Swinden have been co-opted to the Association Committee.

TELEVISION IN SCHOOLS

Mrs. Ross Williamson, School Broadcasting Council Education Officer for the South East, gave an illustrated talk on *Television in Schools* at a meeting of teachers at Dartford Grammar School on Tuesday, 12th March, 1957.

Developments in the preparation of the B.B.C.'s Experimental Television Service for Schools.

The School Broadcasting Council and the B.B.C. consider Television programmes for schools to be a rational extension of sound broadcasting providing a further method of communication—vision. Sound broadcasts for schools are accepted as a valuable educational supplement though at first they were looked on by some with suspicion.

To-day in the educational world there are two main points of view about Television for schools. One is that of a headmaster of a famous school, "Over my dead body!"; the other is that it would be foolish not to make use of this modern invention provided it is used to communicate material of value which supplements school education. As with sound broadcasts, the B.B.C. will provide television programmes for schools at the request of and in co-operation with the Schools Broadcasting Council.

Half the population of the British Isles now has access to a Television set and it is evident that the medium has a special attraction for children. It is surely important that we make use of this medium in schools.

A pilot experiment was carried out in some Middlesex schools in 1952, but before a main experiment could be planned a period of economic recession had to be weathered and three further problems subsequently overcome. First, the provision of receivers to schools—the Local Education Authorities have given assurances that over 350 schools will be equipped with suitable receivers; second, the training of programme producers—four have been chosen who have had teaching experience and are being given training in television—and third, the provision of studios by the B.B.C. By June it is expected that preliminary plans for the autumn will be complete.

The new service starting in the week beginning September 23rd will have three programmes weekly and a repeat of one. The initial series will be:—

(a) Current Affairs (13—15 years) Topical news and background to contemporary life.

(b) General Science (12—14 years) Natural phenomena and social implications of science (To be repeated).

(c) A series on the British Commonwealth of Nations for 11—15 year olds but mainly directed to 11—13's.

The programmes will last 20—25 minutes and will be transmitted between 2 and 2.30 p.m.

Further information about the content of the programmes will be available in July, and for series needing them teachers notes will be issued a few weeks before screening. Schools intending to view should indicate their interest in the appropriate column of the Registration Card sent out by the School Broadcasting Council to *all* schools in June.

The L.E.A.'s Associations, with the assistance of the School Broadcasting Council, have helped to ensure that money used for the purchase and installation of

Television receivers in schools should be spent wisely. In autumn 1956 a panel was set up to conduct appropriate tests and their report "School Television Broadcasting" may be obtained from the Secretary, The Association of Education Committees, 10 Queen Anne Street, London, W.1. It contains valuable information on equipment, viewing conditions, classroom arrangement and includes a list of approved receivers.

There is much to learn about the use of the television medium and this new venture will, it is hoped, make a significant contribution to our knowledge of the role of television in schools and will, undoubtedly, benefit the whole Television service.

Production in the Television studio

With the aid of slides Mrs. Ross Williamson gave a brief but informative picture of how many sides of Television production culminate in the broadcast programme. The heavy expense of even a short programme showed the necessity for good planning at all stages.

The talk ended with the showing of some recorded extracts from recent B.B.C. productions, illustrating the power of the documentary, the ease of explanation by animation and the value of the closeup in experimental science.

In the discussion that followed the speaker expressed the opinion that some of the characteristics distinguishing Television from film were: its immediacy and topicality; its ability to present a series and to offer material planned at the right pace for a single viewing; and its capacity to respond quickly by adjustments in programmes to audience reaction.

The programmes will be assessed in the light of reports from viewing schools and from the Council's Education Officers. At all times the producers must be in close touch with the audiences' reactions and needs. The first two years would be considered a time for experiment and the co-operation of the viewers was essential.

Comment

The teacher's own personality and contact with pupils is the only universal aid to teaching. Books, blackboard and chalk, models, records, radio, filmstrips, films, tape-recorders and many other media have been accepted by teachers as useful in the teaching situation. These aids are only successful when they achieve something that the teacher himself cannot achieve or at least achieve so well.

For the assessment of a new aid, the first question that must be answered is: What can it accomplish that the teacher and other established aids cannot?

Topicality—seeing things happen where they happen when they happen—the speciality of TV. has much to offer in the study of current affairs and allied subjects. What else can this new medium give to the teachers and pupil?

One answer is, that places, persons, skills and minds to which there is a limited access could be widely given to children and students. Another is, that the ability to link both sides of the TV. screen could make possible actual and abstract pupil participation. The utilization of these two aspects suggests applications of TV. to most subjects.

Finally consider mathematics, the foundation of all modern technological developments, its unenterprising teaching and lack of relation to present and future needs is a challenge of today. Can TV. help teachers, as well as their pupils, to master mathematics and meet this challenge?

What do you think?

IAN HARRIS.

"MATHEMATICAL PIE" WALL CHARTS

The first of a series of wall charts on many topics has just been issued by Mathematical Pie. This deals with *Time Chart No. 7* (1550-1585) of the series published in that booklet. Each member of the series is 20″ × 11″ mounted on thick mill board, varnished and eyeletted to hang on classroom walls. The price, including packing and postage, is 6/6 each, and they may be obtained from 97 Chequer Road, Doncaster. In due course the following will become available: *Beginnings of Astronomy No 1, Famous Mathematical Books No 2, Famous Mathematicians No 4, etc.*

The Secretary has received from Mr. F. C. Clark particulars of a *Multi-Purpose Anglemeter* for practical mathematics in schools. The apparatus comprises a baseboard having on its front surface a squared grid, a 180 degree circular arc graduated in degrees, and an outer circular arc extending over a quarter of a circle. A sighting tube is mounted on it and the apparatus also enables the sine, cosine and tangent of angles to be read off to the third place of decimals. The anglemeter is sold by A. Bence and Co., 66 Wood Ride, Orpington, Kent and the price is £1 1s. 0d.

The Association has recently had its first exchange of correspondence with Russia. As a result, Professor A. Markoushevitch has sent us German translations of two small booklets, written by him, which are part of a series intended as self-study reading for students to supplement their normal class instruction. They are interesting because we have nothing like enough books of this kind in England.

Recurring Series, however, gives a rather conventional account of the subject and is lacking in any suggestion of the practical applications which give it life: but *Noteworthy Curves*, illustrated with many diagrams, is indeed the type of book which a Sixth Form student could read on his own with great benefit. Most of the familiar properties of conics are given, together with a number of less well-known properties of lemniscates and cycloids.

This may lead to an exchange of further material, and the Secretary would like to hear from anyone who has a reading knowledge of Russian and who would be prepared to translate incoming correspondence.

BOOK REVIEWS.

Problems in Mathematical Education (1956)

A Report of the Educational Testing Service, Princeton, U.S.

This is a report of a serious survey of the position in the United States. It involved the examination of 250 books and articles, the answering of questionnaires by 86 experts, and the observation of 36 elementary level classes, and 24 secondary.

The report opens by stating that 68% of pupils have ceased to study mathematics by the end of the second year of the secondary stage, and by admitting that no psychological studies that have so far been considered give any sure explanation of the nature of mathematical learning. The despair of finding an adequate solution is reflected by the inclusion of a remark that some have even wondered if ability

in mathematics is merely a refuge for the anti-social elements of the community.

Seventy per cent of the teachers in elementary schools were found to have "a long-standing hatred of arithmetic." One study of the competence of teachers drew the conclusion that even high school teachers "for the most part are ignorant of the mathematical basis of arithmetic."

The report deplores that the curriculum has changed so little in the last hundred years. The way to make the curriculum flexible enough to give scope for the brighter pupils is also seen as a difficulty, and it is acknowledged that an important problem arises here because of the inadequate teacher's fear of the children going beyond her power to deal with the situation.

The urgent need for experimental research in the classroom is recognised, but the depressing admission is made, as the conclusion to the report: "it will take a generation, perhaps two, to make a sizable dent in the problems of mathematical education."

R. M. FYFE.

A First Geometry—D. N. Straker, Parts 1 and 2.

(George Harrap & Co., Ltd.) 6/- each.

"It is the author's contention that children trust their eyes rather than their heads." So says the preface of these new books and on this assumption are they based. Visions of good, appealing, coloured illustrations immediately leap to one's mind but by the time Chapter 1 is encountered all that remains are the usual line diagrams that we have seen in dozens of textbooks for years and years. How can, for example, a circle in printer's ink on page 11 with some shorter lines drawn inside be seen as a sphere? The eyes do not see a sphere and a child's trust in sight would not fulfil the author's purpose. In fact it is difficult to answer the question as to why the books were considered necessary. True, it is probably yet another man's story, inevitably containing variations of presentation, order and language. But, surely, this is not enough.

If the book were issued to pupils of Preparatory Schools in readiness for the Common Entrance Examination—this is its avowed intention—I can well imagine them being dutifully made to work through the copious explanations from early definitions to areas, cyclic quadrilaterals and Pythagoras, presented formally on the whole; but I cannot see them, from this, gleaning any of the excitement, the stimulation, the eagerness for ideas which a First Geometry should give.

Some of the language is too loosely used; lines are measured rather than their lengths, an angle *is* 60°, a scalene triangle has "none of its sides equal"; vertical lines are considered to be parallel, and "angles in a straight line together equal 180°."

Finally I must take issue with the author's statements that his assumptions "would horrify some mathematicians. 'Nothing may be assumed without proof,' they say." Is not the whole structure of mathematics based on assumptions—any assumptions so long as they are consistent within themselves—which, by definition, not only do not require proof but often cannot be proved? Children have a deep-seated need for precise understanding which we should recognise before the age of innocence gives way to sophistication and second best.

JOHN V. TRIVETT.

ASSOCIATION FOR TEACHING AIDS IN MATHEMATICS

CONSTITUTION

1. The name of the Association shall be "THE ASSOCIATION FOR TEACHING AIDS IN MATHEMATICS."

2. The AIMS of the Association shall be to promote the study of the teaching of mathematics, and to improve the aids used therein, through week-end courses, exhibitions, demonstration lessons, and the publication of a bulletin.

3. MEMBERSHIP of the Association is open to all who are interested in these aims.

4. The SUBSCRIPTION shall be five shillings per annum, covering the calendar year, and shall be payable to the Treasurer from the beginning of the year. Subscriptions of new members joining after 1st October shall also cover membership for the following calendar year.

5. The COMMITTEE shall consist of a Chairman, Secretary, Treasurer and nine other members, and shall be responsible for all day to day management of the Association. It shall have power to co-opt additional members, to appoint assistants to Secretary and Treasurer, an Editorial staff for the bulletin, a Librarian, and to delegate responsibility to sub-committees set up to act for particular aspects of the work of the Association.

6. The Committee of the Association shall call an ANNUAL GENERAL MEETING once in the course of each calendar year, and at this Meeting the Committee for the subsequent year shall be elected. The Annual General Meeting shall also have power to fill the office of President for the ensuing year and to elect Vice-Presidents. Such Officers are to be ex-officio members of the Committee. Nominations, duly proposed and seconded and with the consent of the nominee, must reach the Secretary, in writing, at least 28 days before the date of the Annual General Meeting.

7. EXTRAORDINARY GENERAL MEETINGS may be called at any time, either by the Committee, or at the written request to the Secretary of at least twenty members of the Association, with reasonable time allowed for making the necessary arrangements.

8. A QUORUM for a General Meeting shall be 20 members.

9. The CONSTITUTION may be amended by a resolution passed by a simple majority at an Annual General Meeting. Notice of any such resolution must reach the Secretary in writing at least 28 days before the date of the Meeting.

ASSOCIATION FOR TEACHING AIDS IN MATHEMATICS

The Annual Subscription to the Association is 5s.; cheques should be made payable to "A.T.A.M", and should be addressed to the Treasurer. This journal is supplied free to members.

Contributions to the journal should be sent to the Executive Editor. Contributors are asked to keep mathematical notation as typographically simple as they can, and to submit illustrations, where necessary, in a form suitable for reproduction. Enquiries about distribution should be addressed to the Secretary.

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